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ACCURACY OF THE TRIA3 THICK SHELL ELEMENT

William. R. Case, Marco Concha and Mark McGinnis
NASA/Goddard Space Flight Center

SUMMARY

The accuracy of the new TRIA3 thick shell element is assessed via comparison with a theoretical solution for thick homogeneous and honeycomb flat simply supported plates under the action of a uniform pressure load. The theoretical thick plate solution is based on the theory developed by Reissner and includes the effects of transverse shear flexibility which are not included in the thin plate solutions based on Kirchoff plate theory. In addition, the TRIA3 is assessed using a set of finite element test problems developed by the MacNeal-Schwendler Corp. (MSC). Comparison of the COSMIC TRIA3 element as well as those from MSC and Universal Analytics Inc. (UAI), for these test problems is presented. The current COSMIC TRIA3 element is shown to have excellent comparison with both the theoretical solutions and also those from the two commercial versions of NASTRAN with which it was compared.

INTRODUCTION

The TRIA3 thick shell element was added to the 1990 release of COSMIC NASTRAN. Along with the QUAD4, the two new shell elements represent a significant increase in the capability of COSMIC NASTRAN to model complicated shell structures. The deficiencies of the original TRIA1,2 and QUAD1,2 shell elements have been recognized for years and have been reported in the literature. At the Goddard Space Flight Center (GSFC), the triangular and quadrilateral shell elements are used in virtually all structural analyses of our spacecraft and related hardware. Typical applications are for the modeling of cylindrical shells and flat plates made of honeycomb or machined, lightweighted, metal that make up the structure of spacecraft and scientific instruments. In some cases these models require that the effects of transverse shear flexibility be included due to their thickness. The TRIA3 and QUAD4 elements include these effects. The QUAD4 element has, in addition, an improved membrane capability for in-plane loading. The TRIA3 element, due to it's limited number of degrees of freedom retains the constant strain membrane capability of the older TRIA1 and TRIA2 elements. This necessitates finer meshes for in plane loading cases than would be required when using the QUAD4 element.

The purpose of the study reported herein is to assess the accuracy of the TRIA3 element in modeling a variety of situations involving both solid cross-section plates as well as those constructed of honeycomb. An identical study for the QUAD4 element was reported in the 18th NASTRAN User's Colloquium and is documented in reference 1. As with the QUAD4 study, the three goals of the TRIA3 study were to determine:

a) what is the rate of convergence to the theoretical solution as the mesh is refined

b) whether the element exhibits sensitivity to aspect ratios significantly different than 1.0

c) how the element behaves in a wide variety of modeling situations, such as those included in the MSC element test library (discussed below).

The first two questions were addressed in the same manner as several other studies reported by one of the authors in prior NASTRAN colloquia (references 1 - 3). The procedure used in those studies, and followed here also, is to isolate the effects of mesh refinement and aspect ratio. That is, the mesh refinement study is done using elements with an aspect ratio of 1.0. Then, once a fine enough mesh has been reached such that the errors are small, the effects of aspect ratio can be investigated by keeping the mesh the same (i.e. same number of elements) and varying the overall dimensions of the problem, thus resulting in each element aspect ratio changing. Obviously, in order to accomplish this latter step there must be a theoretical solution (or some other equally acceptable comparison solution) to the problem with which to compare the finite element model results. This is needed since, at each step, a problem of different dimensions (and therefore different theoretical solution) is being modeled.

The above tests are important in that they show the rate of convergence toward the theoretical solution as the mesh is refined. Those tests, however, are not sufficient to completely test the accuracy of a finite element since they do not test irregular geometries, or a variety of loadings or material properties. The MSC has developed a comprehensive set of problems for testing finite elements in a variety of situations (reference 4). The library of problems consists of 15 test problems for shell elements that cover all of the parameters mentioned above. This element test library was used to test the TRIA3 element as was done for the QUAD4 element reported in reference 1.

RESULTS OF MESH AND ASPECT RATIO STUDY

For the mesh and aspect ratio study a theoretical comparison solution is highly desirable. Since the effects of transverse shear flexibility are included in the TRIA3 element formulation, a theoretical solution for moderately thick plates, based on Reissner (or Mindlin) thick plate theory is also desirable. Such a solution is given in references 5 and 6 for rectangular simply supported thick plates under the action of a pressure load. Thus, this problem was used for the mesh and aspect ratio portions of the study.

Figure 1 defines the geometry, coordinate system, boundary conditions and loading for the rectangular plate. The thickness indicates a moderately thick plate of length to thickness ratio of 20. The effect of transverse shear flexibility is only approximately 1% on the maximum displacement but is important in discerning the quality of the convergence of the finite element results to the exact theoretical solution. By exact is meant the theoretical basis for the TRIA3 element, which is expressed in the Reissner thick plate theory. Figure 2 shows the finite element mesh geometry used in the mesh and aspect ratio studies. Due to symmetry only one quarter of the plate was modeled. The 4 x 4 mesh shown in figure 2 is an example only; the mesh was varied during the mesh study. However, as was done for all problems, the quad areas were subdivided into triangles in the alternating orientation shown in figure 2.

Figures 3a - 3c show characteristics of the theoretical solution. As indicated in figure 3a the central displacement solution is represented as an infinite series of hyperbolic functions. A FORTRAN computer program was written to compute the theoretical solutions for displacements (using the series shown) as well as stresses (solution not shown). Figures 3b and 3c show the stiffness parameters needed in the theoretical solution for the homogeneous and honeycomb plates. For the honeycomb plate, two different core stiffnesses were investigated. The stiffer one is representative of aluminum honeycomb construction that has been used at the GSFC. The more flexible one was chosen because it represents a core flexibility that is quite low and was expected to be a more critical test of the TRIA3's shear flexibility formulation.

The results of the mesh study, showing the convergence of the TRIA3 solutions to the theoretical, are presented in tabular form in tables 1 - 2 and in graphical form in figures 4 - 7. Both formats show percent error in displacement at the center of the plate as a function of mesh refinement. Results are included for COSMIC 9.0, UAI 11.1 and MSC 66A NASTRAN. The tables merely give exact numbers (along with the theoretical displacements) and the figures contain the same error information, but in graphic form. Figures 4 and 5 and table 1 are the results for the homogeneous plate. The difference between the results in figures 4 and 5 (and that in the two parts of table 1) is that figure 4 (and the top half of table 1) is for a solution in which shear flexibility is included and figure 5 (and the bottom half of table 1) neglects shear flexibility. These two situations were investigated to test the MID3 option on the PSHELL NASTRAN bulk data deck card which allows the effects of shear flexibility to be ignored if MID3 is left blank. As seen in figures 4 and 5 the NASTRAN results converge very rapidly with mesh refinement for COSMIC 9.0, MSC 66A and UAI 11.1. As seen, all versions converge to less than 1% error for a mesh size of 8 x 8.

Figures 6 and 7 and table 2 are the results for the honeycomb plate. Figure 6 (and the top half of table 2) are for the honeycomb plate with the stiffer core and figure 7 (and the bottom half of table 2) are for the more flexible core. As seen in figures 6 and 7 the NASTRAN results for COSMIC 9.0 and the two commercial NASTRAN versions converge very rapidly for the two honeycomb plates as they did for the homogeneous plate.

In order to test the TRIA3's sensitivity to aspect ratio, the model with a 12 x 12 mesh was run in which the plate side dimension in the x direction was varied. This causes the element aspect ratio to vary while maintaining a constant mesh in an attempt to prevent mesh refinement errors from significantly affecting the results. As seen in tables 1 and 2, the TRIA3 results with a 12 x 12 mesh (and aspect ratio of 1.0) have very little error. The results of the aspect ratio study are presented in figures 8 - 10 and tables 3 - 5. Tables 3 - 5 give percent error in the displacement at the center of the plate versus aspect ratio for a model with a mesh of 12 x 12 TRIA3 elements (over one quarter of the plate). As mentioned above, the aspect ratio was varied by changing the dimension of the plate along the x axis. For example, the results for the aspect ratio of 10 are for a plate (and all TRIA3 elements) that is 10 times as long in the x direction as in the y direction. Therefore, the theoretical solution changes with aspect ratio. Figure 8 and table 3 are for the homogeneous plate (with transverse shear flexibility) while figure 9 and table 4 are for the stiff core honeycomb plate and figure 10 and table 5 are for the more flexible core honeycomb plate. Investigation of the percent error in the tables, as well as in figures 8 - 10 show that the TRIA3 has essentially no aspect ratio sensitivity over the range investigated.

Based on the above results, the COSMIC TRIA3 element is seen to give very accurate results for the displacements in the problem investigated, both in comparison to the exact theory and in comparison to the two commercial versions of NASTRAN that we have at the GSFC. Although the results are not presented herein, similarly accurate results were obtained for the shear and moment stress resultants as well. In addition, the rates of convergence for the TRIA3 compare quite favorably with that found for the QUAD4 in reference 1 for this plate bending problem.

RESULTS OF TESTING USING THE MSC ELEMENT TEST LIBRARY

As mentioned earlier, the mesh and aspect ratio studies, while a very useful tool in the evaluation of an element, do not test all of the important variables that affect accuracy in a finite element solution. The MSC element test library mentioned above represents a rather exhaustive series of tests that include many of the element related parameters which affect the accuracy of a finite element solution. Reference 4 gives a detailed description of the test problems along with theoretical answers and the results of the testing on several MSC elements. The reader should consult reference 4 for a complete description of the various problems in the test series. The portion of this series of element tests that relate to shell elements was run by the authors on the TRIA3 elements contained in COSMIC 9.0, UAI 11.1 and MSC 66A. As the MSC does in their report, the results are presented in detail and also in a summary form in which the element is given a letter grade of A through F based on the magnitude of the error. Table 6 shows the summary results for the 15 tests in the series ranging from a simple patch test to modeling of beams (using the TRIA3 element through the depth) and various plates and shells. The meaning of the letter grades is given at the bottom of the table. As pointed out in reference 4, a failing grade for an element in one test is not a reason to dismiss the element. For one thing, the test scores would improve with mesh refinement; the mesh used in most of the problems was quite coarse. Of importance in this discussion is not the actual grades listed in table 6 but the comparison of the COSMIC grades with those from the other two programs. As seen in table 6, the COSMIC TRIA3 element is as good as, or better than, those of the commercial programs. All of the low marks (D or F) are apparently due to the constant strain membrane portion of the TRIA3 element and the low order mesh used in those problems. For example, the straight beam bending, with in-plane loading, had only one TRIA3 through the thickness. This was done to keep the same mesh as MSC used for the QUAD4 element tests, and was also done in reference 1. Refining the mesh would have improved the answers to any degree of accuracy desired; the low grades are not indicative of any failure of the element to converge. Although not shown in table 6, the old TRIA2 element (included in reference 4) has a D or F grade in 9 of the 15 problems. The twisted beam test (number 11 in table 6) is really used to test the effect of warp on quadrilateral elements, which is not applicable for the TRIA3 element.

CONCLUSIONS

The COSMIC TRIA3 general purpose flat shell element has been shown to be an excellent element and, together with the QUAD4 quadrilateral flat shell element, significantly enhances the usefulness of COSMIC NASTRAN. The element has been shown to compare excellently with those available in two commercial versions of NASTRAN that are currently being used at the GSFC.

REFERENCES

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List of Symbols

w = plate displacement in the z direction
x,y,z = coordinate directions
p = pressure load on the plate in the z direction
a, b = plate dimensions (length, width)
t = overall plate thickness
D = plate bending rigidity (see Figures 3b, 3c)
Cs, C_n = plate shear stiffness (see Figures 3b, 3c)
t_f = thickness of face sheets for honeycomb plate
t_c = thickness of the core for honeycomb plate
N_x = number of elements in x direction in one quarter of plate
N_y = number of elements in y direction in one quarter of plate
AR_e = element aspect ratio (see Figure 2)
E = Young's modulus
G = shear modulus
ν = Poisson's Ratio

TABLE 1: TRIA3 Error in Displacement at Center of Plate

Mesh Size Study (Element Aspect Ratio 1.0)

Simply-Supported, Homogeneous Plate Under Uniform Pressure Load

Theoretical Displacements

With Transverse Shear Flexibility: 3.571×10^{-5} m (1.406×10^{-3} in.)

Without Transverse Shear Flexibility: 3.529×10^{-5} m (1.390×10^{-3} in.)

Mesh	Cosmic 90	% Error UAI Ver. 11.1	MSC Ver. 66A
With Transverse Shear Flexibility			
1x1	39.33	27.64	16.62
2x2	13.63	11.36	9.01
4x4	3.29	2.77	2.06
8x8	0.01	0.55	0.34
12x12	0.00	0.13	0.04
Without Transverse Shear Flexibility			
1x1	40.56	28.37	17.45
2x2	14.31	11.72	9.52
4x4	3.74	3.01	2.43
8x8	0.95	0.76	0.62
12x12	0.42	0.34	0.27

TABLE 2: TRIA3 Error in Displacement at Center of Plate

Mesh Size Study (Element Aspect Ratio 1.0)
Simply-Supported, Honeycomb Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

Theoretical Displacements

$G_Z = 1.517\text{x}10^8$ N/m^2 : $2.422\text{x}10^{-3}$ m ($9.535\text{x}10^{-2}$ in.)

$G_Z = 1.379\text{x}10^7$ N/m^2 : $3.102\text{x}10^{-3}$ m ($1.221\text{x}10^{-1}$ in.)

Mesh	Cosmic 90	% Error UAI Ver. 11.1	MSC Ver. 66A
$G_Z = 1.517\text{x}10^8$ N/m^2 (22000 psi)			
1x1	38.28	27.13	16.08
2x2	13.36	11.36	8.88
4x4	3.35	2.92	2.16
8x8	0.81	0.73	0.52
12x12	0.37	0.33	0.24
$G_Z = 1.379\text{x}10^7$ N/m^2 (2000 psi)			
1x1	24.07	17.82	7.37
2x2	9.71	8.83	6.35
4x4	2.48	2.26	1.60
8x8	0.60	0.55	0.38
12x12	0.30	0.28	0.02

TABLE 3: TRIA3 Error in Displacement at Center of Plate

Aspect Ratio Study (12 x 12 Mesh)

Homogeneous, Simply-Supported Plate

Under Uniform Pressure Load with Transverse Shear Flexibility

AR	theoretical w, m (in.)	% Error Cosmic 88	% Error UAI Ver. 10.0	% Error MSC Ver. 65C
1	3.571×10^{-5} (1.406×10^{-3})	0.17	0.13	0.04
2	8.865×10^{-5} (3.490×10^{-3})	0.14	0.10	0.03
5	11.34×10^{-5} (4.465×10^{-3})	0.11	0.11	0.07
10	11.38×10^{-5} (4.482×10^{-3})	0.08	0.08	0.05

TABLE 4: TRIA3 Error in Displacement at Center of Plate

Aspect Ratio Study (12 x 12 Mesh)

Stiff Core, Simply-Supported, Honeycomb Plate

Under Uniform Pressure Load with Transverse Shear Flexibility

AR	theoretical w, m (in.)	% Error Cosmic 88	% Error UAI Ver. 10.0	% Error MSC Ver. 65C
1	2.422×10^{-3} (9.535×10^{-1})	0.37	0.33	0.24
2	5.974×10^{-3} (2.352×10^{-1})	0.28	0.24	0.17
5	7.631×10^{-3} (3.004×10^{-1})	0.21	0.21	0.17
10	7.660×10^{-3} (3.016×10^{-1})	0.21	0.21	0.17

TABLE 5: TRIA3 Error in Displacement at Center of Plate

Aspect Ratio Study (12 x 12 Mesh)

Flexible Core, Simply-Supported, Honeycomb Plate

Under Uniform Pressure Load with Transverse Shear Flexibility

AR	theoretical w, m (in.)	% Error Cosmic 90	UAI Ver. 11.1A	MSC Ver. 66A
1	3.102×10^{-3} (1.221×10^{-1})	0.30	0.28	0.20
2	7.026×10^{-3} (2.766×10^{-1})	-0.67	0.24	0.18
5	8.785×10^{-3} (3.459×10^{-1})	0.22	0.22	0.17
10	8.815×10^{-3} (3.470×10^{-1})	0.17	0.17	0.13

TABLE 6

SUMMARY OF TEST RESULTS FOR TRIA3 SHELL ELEMENTS

Test	Elem. Loading		Element Shape	COSMIC 90	UAI 11.1A	MSC 66A
	In Plane	Out of Plane				
1. Patch Test	X		Irregular	A	A	A
2. Patch Test		X	Irregular	A	A	A
3. Straight Beam, Extension	X		All	A	A	A
4. Straight Beam, Bending	X		Regular	F	F	F
5. Straight Beam, Bending	X		Irregular	F	F	F
6. Straight Beam, Bending		X	Regular	B	B	B
7..Straight Beam, Bending		X	Irregular	B	B	B
8. Straight Beam, Twist			All	F	F	F
9. Curved Beam	X		Regular	F	F	F
10. Curved Beam		X	Regular	F	F	F
11. Twisted Beam	X	X	Regular	C	C	D
12. Rectangular Plate (N=4)		X	Regular	B	B	B
13. Scordelis-Lo Roof (N=4)	X	X	Regular	D	D	D
14. Spherical Shell (N=8)	X	X	Regular	A	A	A
15. Thick-Walled Cylinder (nu=.4999)	X		Regular	A	A	A
Number of Failed Tests (D's and F's)				6	6	7

Grading for Shell Element Test Results

Grade	Requirement
A	2% ≥ Error
B	10% ≥ Error > 2%
C	20% ≥ Error > 10%
D	50% ≥ Error > 20%
F	Error ≥ 50%

Fig. 1
Test Problem

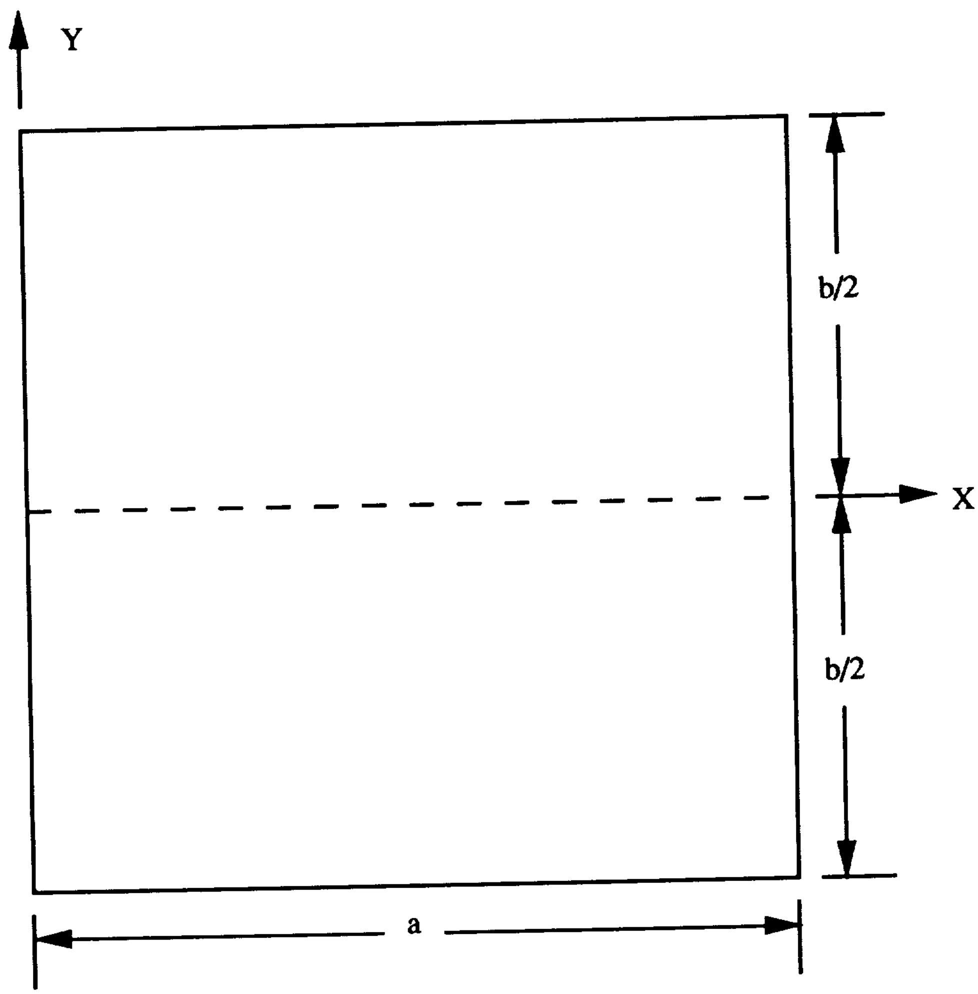
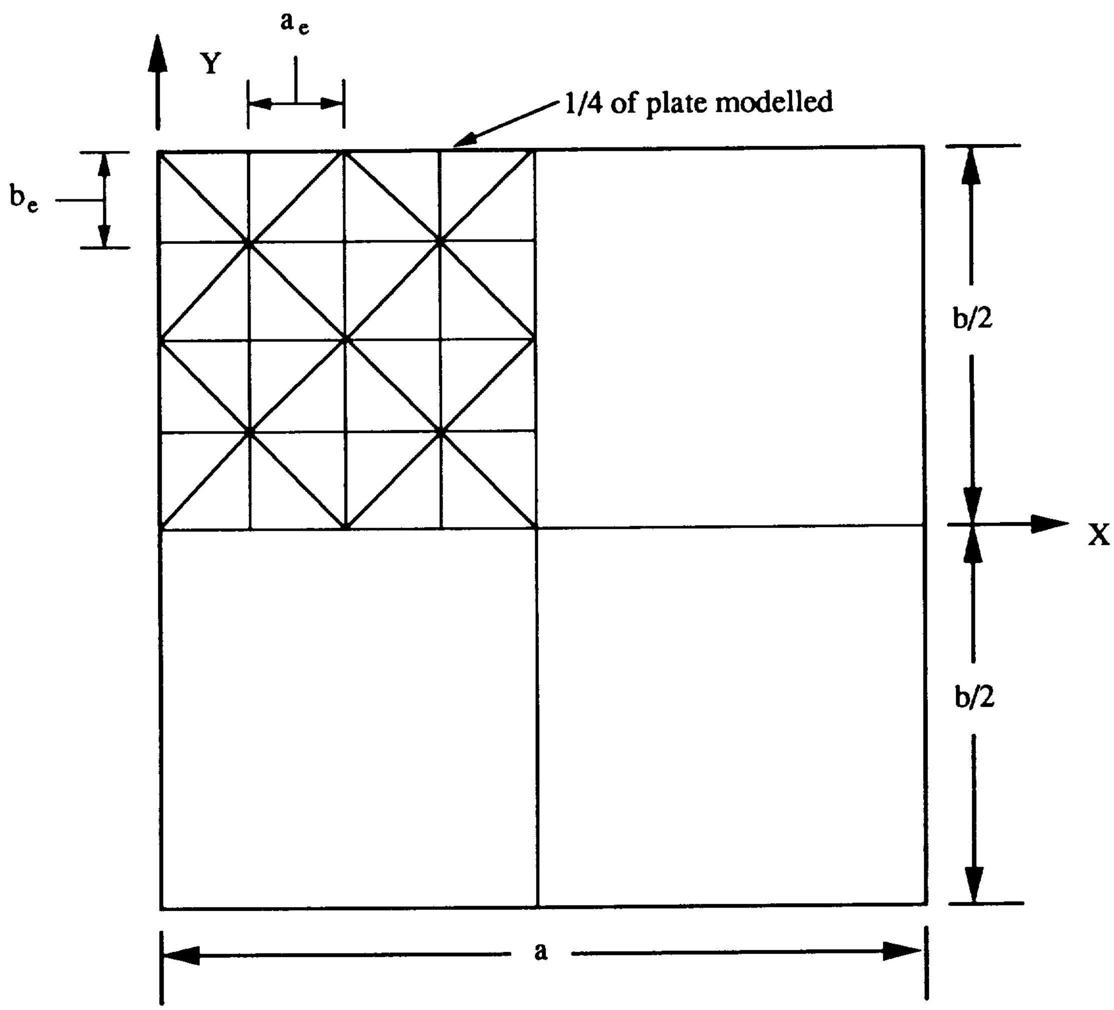


Plate Size: a=1.016 m (40. in.)* b=1.016m (40.in.)
Boundary Conditions: simply supported on all edges
Loading: pressure load, p=6895. N/m^2 (1.0 psi) +Z direction
Thickness: t=0.0508 m (2.0 in.)

*: Variable in aspect ratio studies

Fig. 2
Mesh Geometry



$AR_e = a_e/b_e$ = element aspect ratio
$N_x = a/2a_e$ = number of elements in X direction in 1/4 of plate
$N_y = b/2b_e$ = number of elements in Y direction in 1/4 of plate

Fig. 3a

Theoretical Solution - Central Displacement

Central Displacement

$$w\left(x=\frac{a}{2}, y=0\right) = \frac{4p}{aD} \sum_{m=1,3,5,\ldots} \left[1 + C_5 \cosh(\mu y) + \mu y C_6 \sinh(\mu y) + \mu^2 D\left(\frac{1}{C_s} - \frac{1+\nu}{C_n}\right)\right] \frac{\sin \mu x}{\mu^5}$$

where,

$$C_5 = -\frac{1}{\cosh \alpha_m}\left[1 + \mu^2 D\left(\frac{1}{C_s} - \frac{1+\nu}{C_n}\right) + \frac{1}{2}\alpha_m \tanh(\alpha_m)\right]$$

$$C_6 = \frac{1}{2\cosh \alpha_m}$$

$$\alpha_m = \frac{m\pi}{2}\frac{b}{a}, \quad \mu = \frac{m\pi}{a}$$

Fig. 3b

Theoretical Solution - Homogeneous Plate Parameters

Homogeneous Plate

$$D = \frac{Et^3}{12(1-\nu^2)}$$

$$C_n = \frac{5}{6}\frac{Et}{\nu}$$

$$C_s = \frac{5}{6}Gt, \quad G = \frac{E}{2(1+\nu)}$$

$E = 6.89 \times 10^{10}$ N/m^2 (10.0×10^6 lb/in^2)

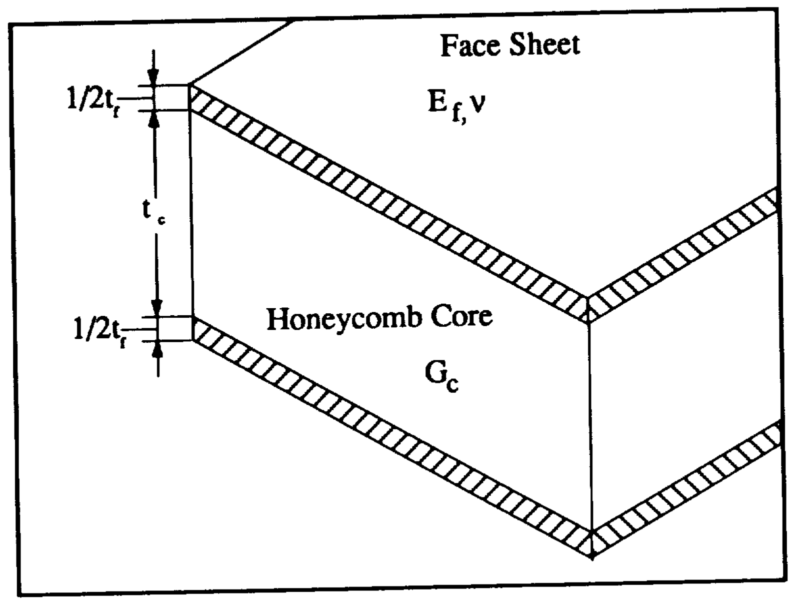
$\nu = 0.33$

$t = .0508$ m (2.0 in.)

Fig. 3c

Theoretical Solution - Honeycomb Plate Parameters

Honeycomb Plate



Core Detail

$$D = \frac{E_f t_f (t_c + t_{f/2})^2}{4(1-\nu^2)}$$

$C_n = \infty$

$C_s = t_c G_c$

$E_f = 6.89 \times 10^{10}$ N/m^2
(10×10^6 lb/in^2)

$\nu = 0.33$

$G_c = 1.379 \times 10^7$ N/m^2 (2000. lb/in^2) Flexible Honeycomb Plate
or
1.517×10^8 N/m^2 (22000. lb/in^2) Stiff Honeycomb Plate

t_c = .0508 m (2.0 in)

t_f = .254 mm (.01 in.)

$t = t_c + t_f$

Fig. 4

TRIA3 Error in Displacement at Center of Plate

Mesh Size Study (Element Aspect Ratio 1.0)
Simply-Supported, Homogeneous Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

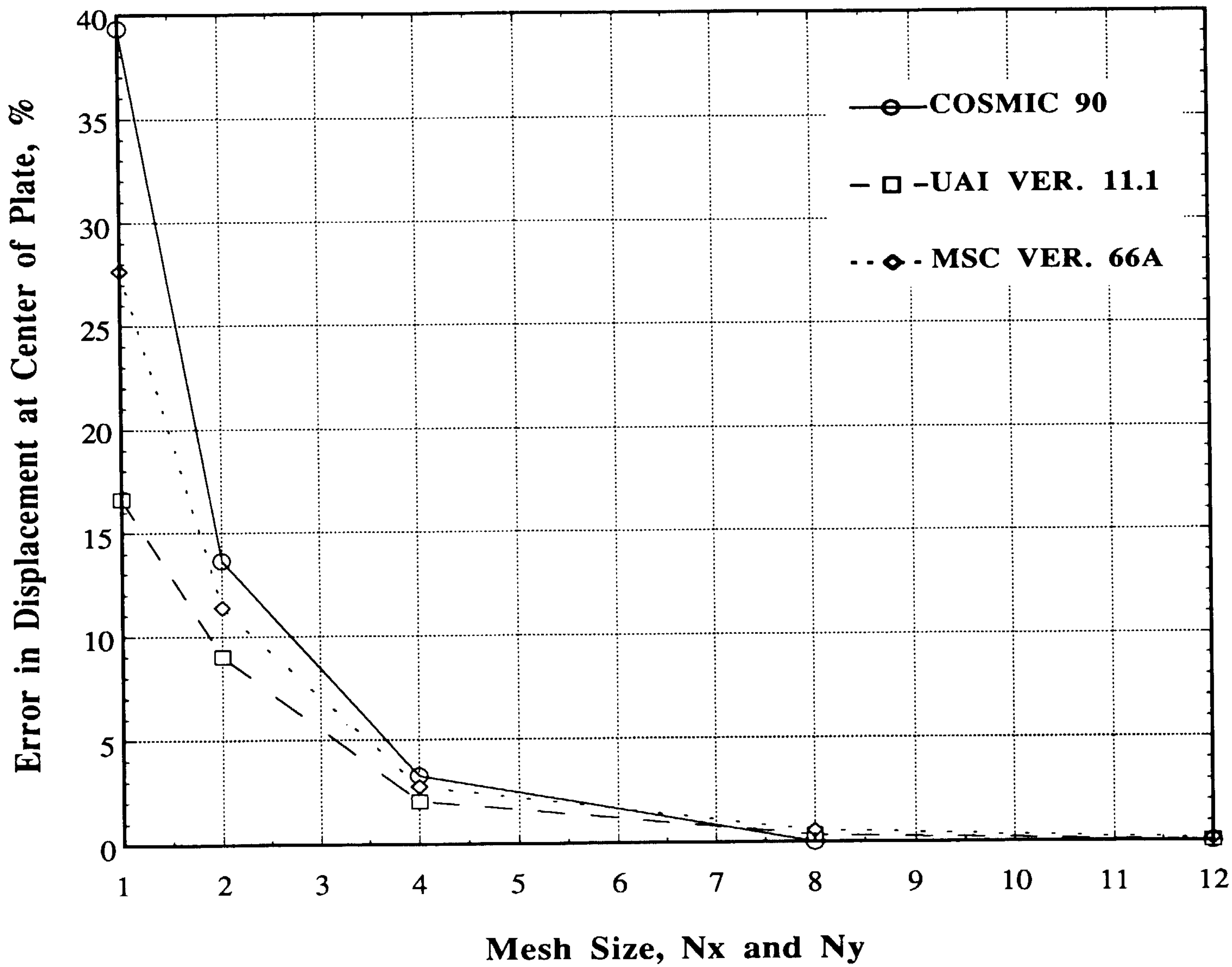


Fig. 5

TRIA3 Error in Displacement at Center of Plate

Mesh Size Study (Element Aspect Ratio 1.0)

Simply-Supported, Homogeneous Plate

Under Uniform Pressure Load without Transverse Shear Flexibility

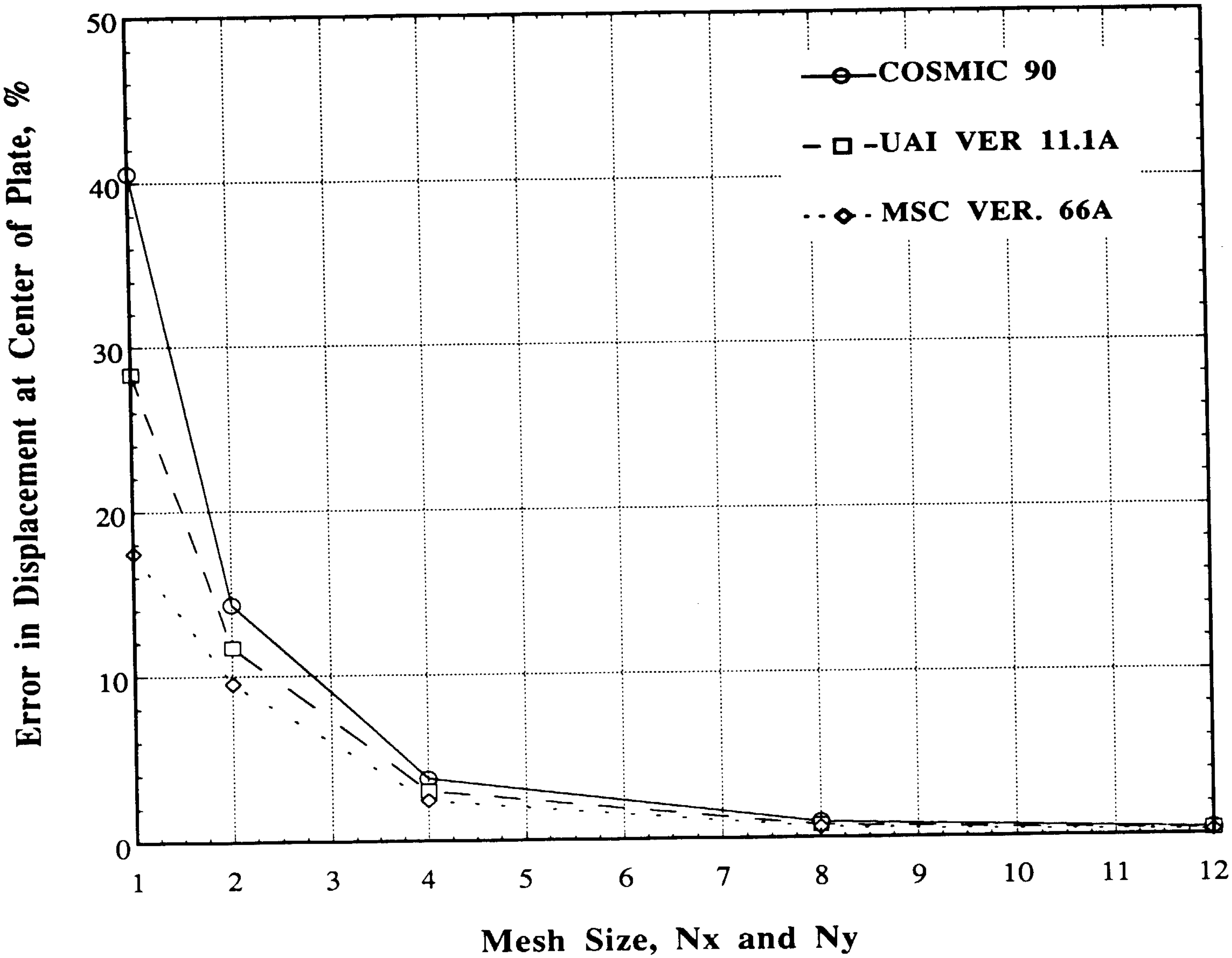


Fig. 6
TRIA3 Error in Displacement at Center of Plate
Mesh Size Study (Element Aspect Ratio 1.0)
Stiff Core, Simply-Supported, Honeycomb Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

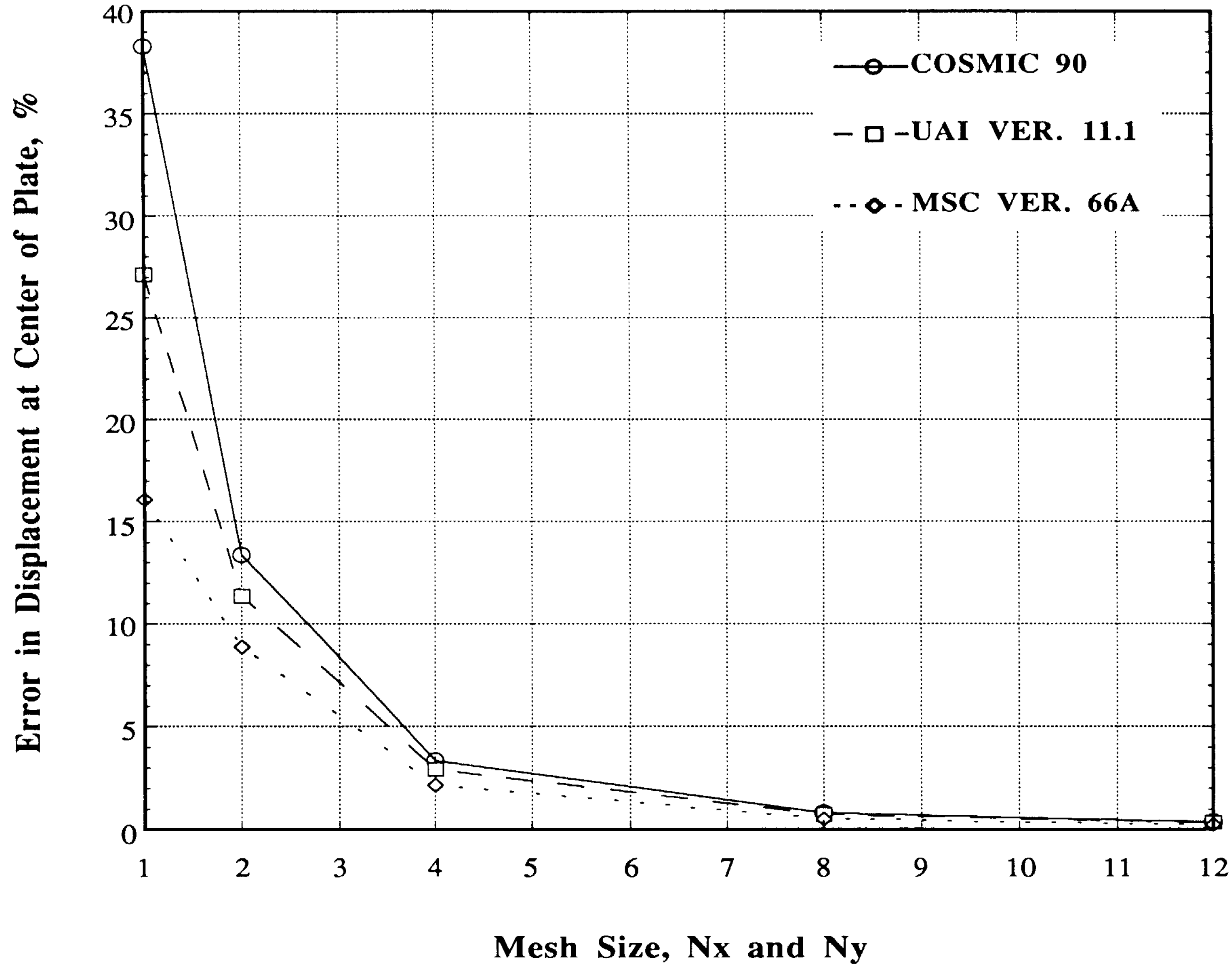


Fig. 7

TRIA3 Error in Displacement at Center of Plate

Mesh Size Study (Element Aspect Ratio 1.0)
Flexible Core, Simply-Supported, Honeycomb Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

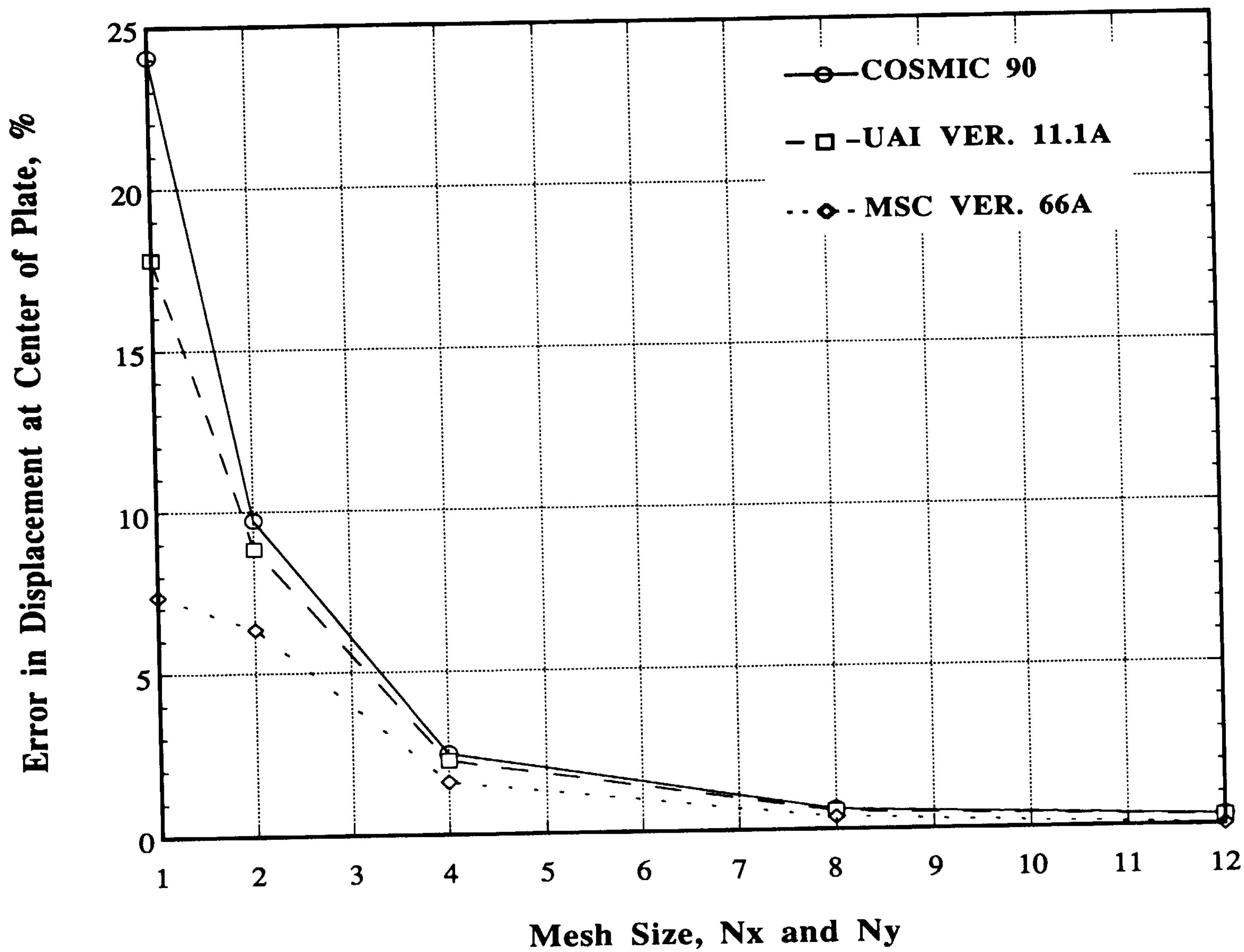


Fig. 8

TRIA3 Error in Displacement at Center of Plate

Aspect Ratio Study (12 x 12 Mesh)
Homogeneous, Simply-Supported Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

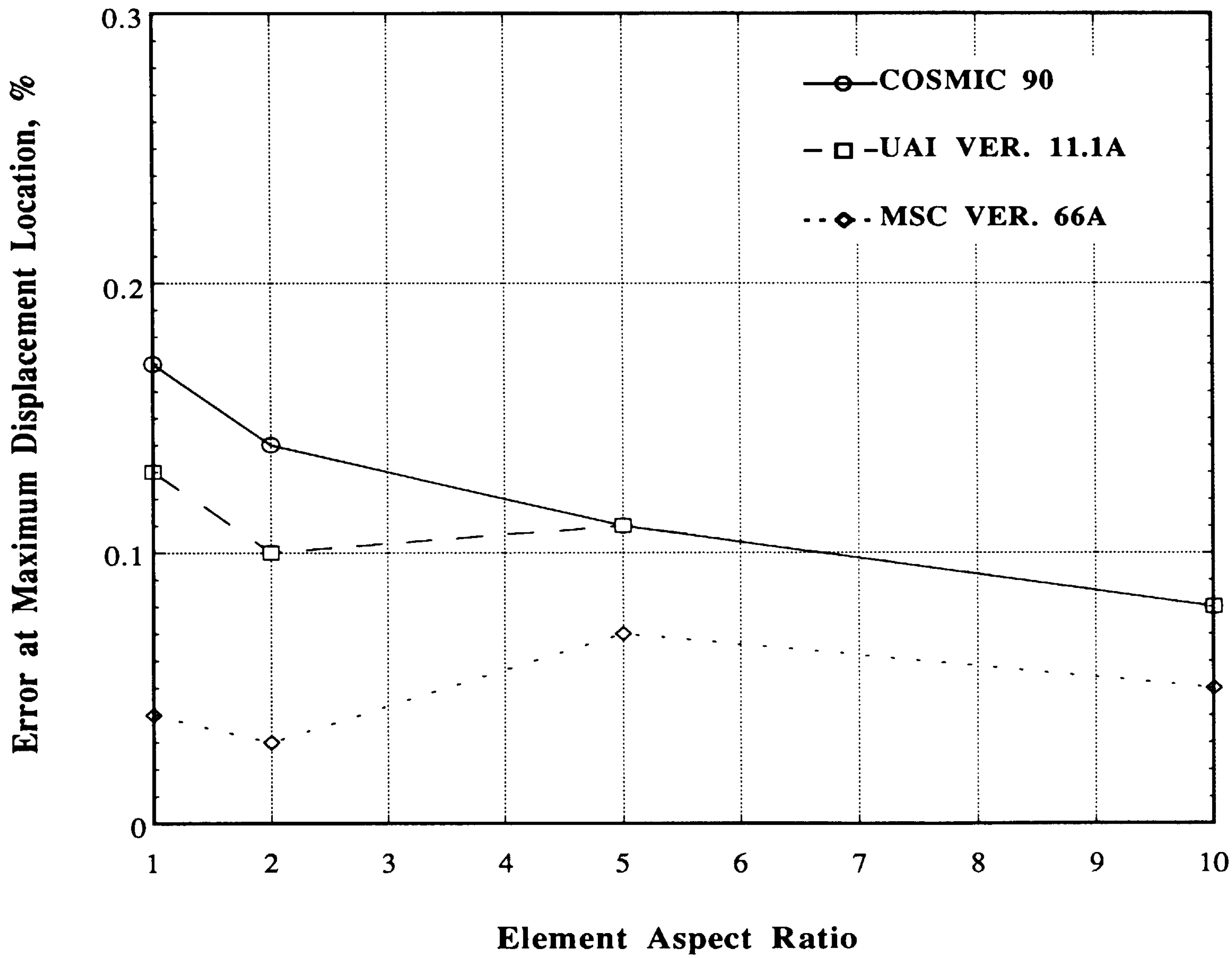


Fig. 9
TRIA3 Error in Displacement at Center of Plate
Aspect Ratio Study (12 x 12 Mesh)
Stiff Core, Simply-Supported, Honeycomb Plate
Under Uniform Pressure Load with Transverse Shear Flexibility

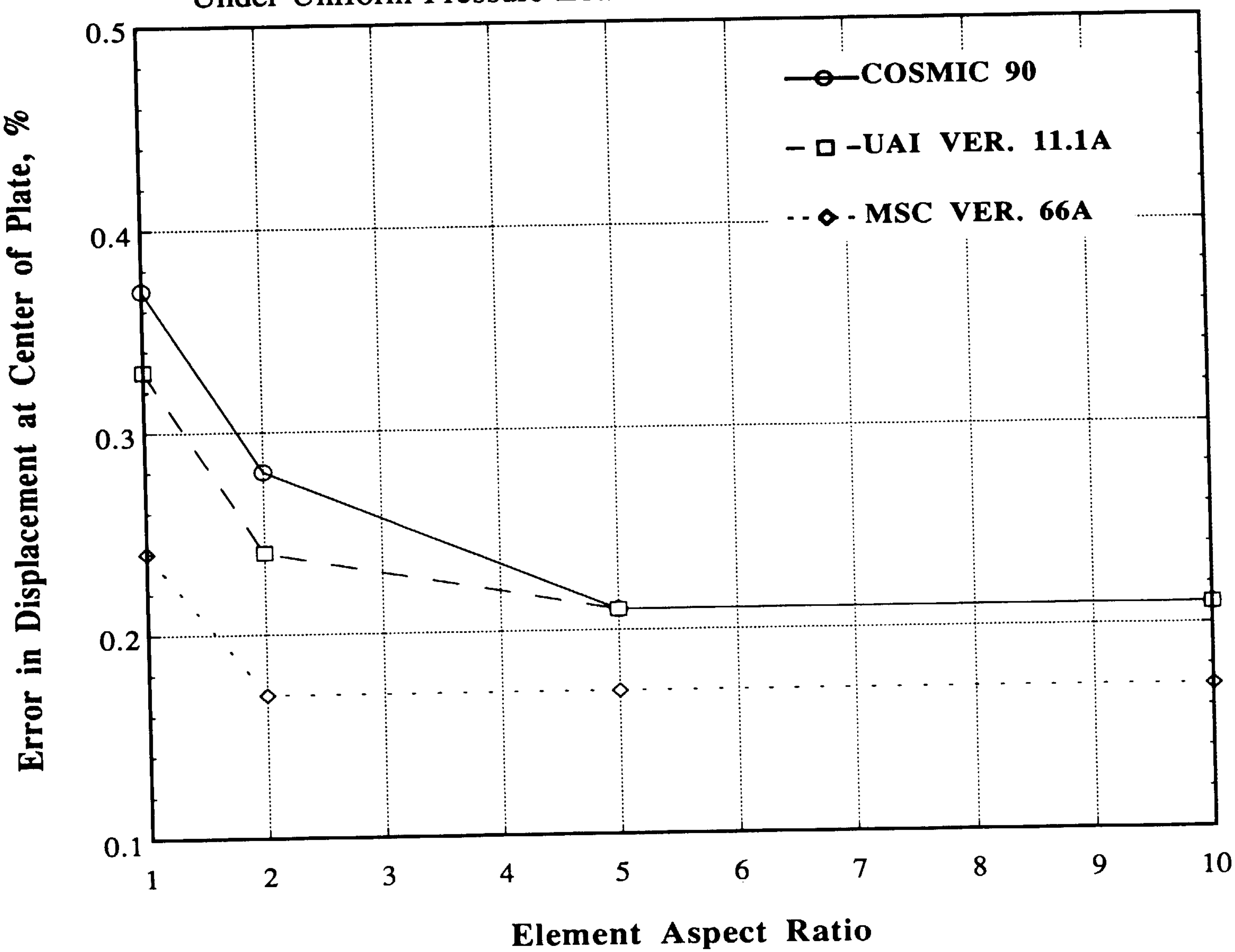


Fig. 10

TRIA3 Error in Displacement at Center of Plate

Aspect Ratio Study (12 x 12 Mesh)
Flexible Core, Simply-Supported, Honeycomb Plate
Under Uniform Pressure Load with Transverse Shear Flexibility